

# The Effect of Abnormal Granulation on Acoustic Wave Travel Times and Mode Frequencies

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Received;accepted

**Abstract.** Observations indicate that in plage areas (i.e. in active regions outside sunspots) acoustic waves travel faster than in quiet sun, leading to shortened travel times and higher  $p$ -mode frequencies. Coupled with the 11-year variation of solar activity, this may also explain the solar cycle variation of oscillation frequencies. While it is clear that the ultimate cause of any difference between quiet sun and plage is the presence of magnetic fields of order 100 G in the latter, the mechanism by which the magnetic field exerts its influence has not yet been conclusively identified. One possible such mechanism is suggested by the observation that granular motions in plage areas tend to be slightly “abnormal”, dampened compared to quiet sun.

In this paper we consider the effect that abnormal granulation observed in active regions should have on the propagation of acoustic waves. Any such effect is found to be limited to a shallow surface layer where sound waves propagate nearly vertically. The magnetically suppressed turbulence implies higher sound speeds, leading to shorter travel times. This time shift  $\Delta\tau$  is independent of the travel distance, while it shows a characteristic dependence on the assumed plage field strength. As a consequence of the variation of the acoustic cutoff with height,  $\Delta\tau$  is expected to be significantly higher for higher frequency waves within the observed regime of 3–5 mHz. The lower group velocity near the upper reflection point further leads to an increased envelope time shift, as compared to the phase shift.  $p$ -mode frequencies in plage areas are increased by a corresponding amount,  $\Delta\nu/\nu = \nu\Delta\tau$ . These characteristics of the time and frequency shifts are in accordance with observations. The calculated overall amplitude of the time and frequency shifts are comparable to, but still significantly (factor of 2 to 5) less than suggested by measurements.

**Keywords:** Sun: faculae, plages — Sun: granulation — Sun: helioseismology

## 1. Introduction

Early observations proved that the global acoustic frequencies show small but significant and systematic correlations with the solar cycle. Although the acoustic modes are strongly evanescent in the atmosphere, changes of magnetic fields and mean temperature at the boundary layer (i.e. lower atmosphere) could play a rather important role in the determination of their frequencies acting as boundary conditions in an eigenvalue problem when determining these frequencies,



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since magnetic or flow fields in the boundary layer change the mean elasticity of the boundary itself or alter the upper turning points. These effects contribute to small corrections to the eigenfrequencies. The pioneering papers by Campbell and Roberts (1989) and Roberts and Campbell (1986), opened a new series of studies of the coupling of solar global oscillations to the lower solar atmosphere. The influence of an atmospheric coherent magnetic field on  $p$ - and  $f$ -mode frequencies was evaluated theoretically for a simple and elegant model. It was shown that, at low to moderate degree  $l$ , an increase in chromospheric magnetic field leads to a frequency *increase* for the  $n = 1$   $p$ -mode, whereas the overtones ( $n=2, 3$ , etc.) suffer a frequency decrease. It was shown that at high  $l$ , all the  $p$ -modes suffer a frequency decrease. The effect of the coherent magnetic canopy on the solar acoustic modes was extended to allow for variations in height of the magnetic canopy (Evans and Roberts, 1991, 1992). It was found that changes in chromospheric magnetism can be manifested in  $p$ - and  $f$ -mode data sets gathered at different phases of the solar cycle. These predictions of solar-cycle variability in high-degree  $p$ -mode frequencies from a simple model of the magnetic canopy which permeates the solar atmosphere were compared with the observations of Libbrecht and Woodard (1990). Good agreement was found with the observed frequency shifts for modes of frequency less than 4 mHz, through a mechanism in which an increasing magnetic field induces “stiffening” of the Sun’s lower atmosphere.

The influence of short-scale motions (i.e. granulation) modelled as a random flow on  $f$ -mode frequencies was first evaluated by Murawski and Roberts (1993a,b; see also Murawski and Goossens, 1993, Ghosh, Antia, and Chitre, 1995, Gruzinov, 1998, and Mędrek, Murawski, and Roberts, 1999). Erdélyi, Kerekes, and Mole (2004, 2005) have revisited this problem. The  $f$ -mode is essentially a surface wave; hence the mode frequencies are less likely to be influenced by the solar stratification. Most probably the discrepancies are the result of near surface mechanisms, such as interactions with surface or sub-surface magnetic fields and flows. Erdélyi, Kerekes, and Mole followed the general approach of Murawski and Roberts, which is a valuable one, but corrected certain errors which appeared in that paper. The simple model used by Murawski and Roberts and Erdélyi, Kerekes, and Mole gives a deviation of the  $f$ -modes from the theoretically predicted parabolic ridges which agrees qualitatively with observations. They found that turbulent background flows can *reduce* the eigenfrequencies of global solar  $f$ -modes by several percents, as found in observations at high spherical degree. Extensive numerical simulations of the outer parts of the Sun carried out by, e.g., Rosenthal *et al.* (1999) demonstrated and quantified the influence of turbulent convection on solar oscillation frequencies.

In the lower part of the solar atmosphere, i.e. in the *boundary layer* between the solar interior and the magnetically dominant solar corona there are both coherent and random components of the velocity and magnetic fields, each of which may contribute to the frequency shifts and line widths of the global solar acoustic oscillations. Before a comprehensive model including all these effects is constructed, it is vital for a better understanding to estimate the importance of these various effects. Random flows (e.g. turbulent granular motion), coherent flows (meridional flows or the near-surface component of the differential rotation), random magnetic fields (e.g. the continuously emerging tiny magnetic fluxes or magnetic carpet) and coherent fields (large loops and their magnetic canopy region) may each affect the acoustic perturbations. Some of these effects may be more important than others. The magnitude of these corrections has to be estimated one by one and it is suspected that, unfortunately, they all may contribute to line widths or frequency shifts of the global acoustic oscillations on a rather equal basis. In what follows we recall briefly some previous findings, in order to have a basis for comparison with the present results.

The magnetic field in the photosphere has, like the granulation, a random component. High resolution magnetograms reveal that outside active regions the solar surface is covered with a mixed polarity network, which has been termed the *magnetic carpet* (Title and Schrijver, 1998). Erdélyi, Kerekes, and Mole (2005) investigated the influence of this disorganised, small-scale lower atmospheric field on the  $f$ -mode frequencies. The magnetic carpet was modelled as a time-independent, stochastic field. Since, depending on their spherical degree, some  $f$ -modes may have a life-time comparable to the characteristic replacement time (of the order of tens of hours) of the magnetic carpet, this limits the validity of the Erdélyi et al. study. Nonetheless they found that a time-independent random magnetic field can significantly *increase* the  $f$ -mode frequencies.

Flow fields at the lower atmospheric boundary layer may also be random or coherent. By inverting the observational data of solar global oscillations one could potentially reconstruct the global flow structures. Large-scale sub-surface flows were found by this technique (e.g. Braun and Fan, 1998). To the best of our knowledge, Erdélyi, Varga, and Zétényi (1999) were the first who studied the effect of a sub-surface motion on magnetoacoustic-gravity surface waves, representing the  $f$ -mode in a model of the solar interior - solar atmosphere interface. They found that the flow causes a shift of the forward and backward propagating magnetoacoustic-gravity modes, which in certain cases bifurcate. Erdélyi & Taroyan (1999, 2001) generalised the model by allowing the temperature to increase linearly with depth in the sub-surface zone.

They derived the dispersion relation and analytical formulae for the frequencies of  $p$ - and  $f$ -modes in the limit of small wave numbers. Numerical solutions were presented for other cases.

All these global studies suggest that the strong magnetic perturbations associated with active regions can have an important effect on the propagation and reflection of sound waves near the solar surface. Time-distance and other types of local helioseismic studies confirm that the travel time of sound waves is significantly different in the subphotospheric layers of active regions than in the quiet Sun. In plage areas, outside sunspots, the travel time is generally found to be shorter than in quiet sun areas (e.g. Chou, 2000; Kosovichev, Duvall, and Scherrer, 2000; Hughes, Rajaguru, and Thompson, 2005).  $p$ -mode frequencies are found to be systematically higher in plage areas than in quiet sun (Rajaguru, Basu, and Antia, 2001). All this implies either a different form of the dispersion relation in plage areas, or a perturbed background stratification (and consequently a perturbed sound speed and acoustic cutoff).

Several possible physical mechanisms have been proposed to explain such discrepancies in magnetized regions. Zweibel and Bogdan (1986) considered the effect that the presence of an ensemble of magnetic flux fibrils in plage regions has on sound wave propagation. Brüggen and Spruit (2000), in turn, discussed the effects an altered temperature profile could have on sound wave travel times. One plausible effect of plage magnetic fields has, however, apparently not been considered from the point of view of acoustic propagation: the magnetic suppression of turbulent motions, observable in the form of “abnormal granulation” (Dunn and Zirker, 1973). The aim of the present work is to complement previous work by considering the potential effect of abnormal granulation on sound wave travel times in plage areas. Note that here we only consider the effect of turbulence on the propagation of coherent waves, disregarding wave excitation and scattering.

Our method of calculation, based on the standard ray approximation and on the assumption of isotropic turbulence, is described in Section 2. Results are presented in Section 3, while Section 4 concludes the paper.

## 2. Method

## 2.1. RAY APPROXIMATION

The ray approximation, a widely used method of time-distance helioseismology, is valid in the short wavelength limit  $\lambda \ll H$ , where  $H$  is the density scale height. As in the solar convective zone the integral scale of turbulence is  $l \geq H$ , it is then consistent to use ray approximation also for the study of wave-turbulence interactions.

The local dispersion relation for a travelling wave of wave vector  $\mathbf{k}$  in the stratified superadiabatic fluid reads

$$\omega^2 = \omega_c^2 + c^2 k^2, \quad (1)$$

where  $c$  is the adiabatic sound speed, and  $\omega_c \simeq c/2H$  is the acoustic cutoff frequency. Our assumption  $\lambda \ll H$  would clearly imply that the wave frequency is high compared to the acoustic cutoff:  $\omega \sim c/\lambda \gg c/H \sim \omega_c$ . On the other hand, the upper reflection of the sound waves, responsible for forming the wave duct, is due to the sudden increase of the acoustic cutoff in the photosphere. Thus, the ray approximation is bound to break down in a shallow layer below the surface: its widespread use is based on the circumstance that near the surface the waves propagate nearly vertically, so their propagation is not expected to be strongly influenced by interference with scattered or reflected waves. In line with this reasoning, the neglect of  $\omega_c$  in the dispersion relation is consistent with the use of the ray approximation. Sound waves then become nondispersive, their upper reflection imposed externally at a prescribed depth  $z_t$  where  $\omega_c = \omega$ . (Indeed, D'Silva, 1996a finds that acoustic wave packets are only very slightly dispersive.) The effect of turbulence on nondispersive waves can then be described by a correction factor  $F_t$ :

$$\omega = F_t c k. \quad (2)$$

In the general case  $F_t$  is anisotropic; however, for the present calculation we consider isotropic turbulence, resulting in a scalar  $F_t$ . The form of  $F_t$  will be discussed in Section 2.2 below. Turbulent convection is certainly anisotropic and inhomogeneous; however, presently very little is known about how this anisotropy differs between abnormal granulation and normal granulation. Thus, for this first exploratory study we consider it more appropriate to limit the number of unknown parameters by restricting attention to isotropic turbulence.

As the magnetic suppression of turbulence is expected to be confined to rather shallow depths, we use a Cartesian setup. Then the ray equations forming the basis of time-distance helioseismology (D'Silva, 1996b) reduce to very simple forms. Introducing the effective sound speed  $\mathbf{C} = F_t c \mathbf{k}/k$ , the equations can be recast as a relation for the

ray path:

$$\frac{dz}{dx} = \frac{C_z}{C_x}, \quad (3)$$

or in integral form

$$x = \int \frac{C_x}{C_z} dz, \quad (4)$$

and an integral expression (taken along the ray path) for the acoustic travel time of a wave packet

$$\tau = \int \frac{ds}{C}. \quad (5)$$

Substituting equation (2) into equations (4)–(5) yields the travel time–distance relation in a parametric form.

## 2.2. MAGNETIC SUPPRESSION OF TURBULENCE

Throughout this paper we assume that the turbulent motions are very subsonic:  $\beta \equiv v/c \ll 1$ . This is a good approximation in subphotospheric layers of the Sun. In this limit Taylor’s “frozen turbulence” hypothesis (Taylor, 1938) applies, i.e. the turbulent flow can be regarded as random in space but stationary in time (over the timescales relevant for sound propagation through the turbulent fluctuations).

While the random fluctuations also lead to a random refraction of the ray path, in homogeneous and isotropic turbulence the *mean* wave path remains straight; thus, the correction factor  $F_t$  in equation (2) is a scalar, and the mean ray propagation is only affected by the statistics of the flow component  $v = \mathbf{v} \cdot \mathbf{k}/k$  parallel to the wave vector; the problem is thus reduced to one dimension.

The effect of homogeneous and isotropic turbulence on wave propagation in the ray approximation is then twofold. Scalar fluctuations in the thermodynamical variables imply fluctuations in the sound speed  $c$ , whereas random flows introduce Doppler corrections to the wavenumber. Ensemble averaging over different realizations of the turbulent flow, neither of these effects will cancel out, as already pointed out by Brown (1984). The reason is simple to understand: in the frozen turbulence limit the wave packet will spend more time in those parts of the flow where the sound speed is below average, or  $v < 0$ . (Note that in the opposite limit  $\beta \gg 1$ , in contrast, any net mean effect of turbulence should cancel out in the ray approximation.)

Let us consider the first of these effects: a straight acoustic ray propagating in a medium with random flows, of PDF<sup>1</sup>  $f(v)$ , aligned with

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<sup>1</sup> The probability distribution function (PDF) is here meant in the sense of the frequency interpretation of probability, i.e.  $f(v) dv$  is the fraction of time when the

the direction of propagation. The mean travel time over unit distance is then clearly

$$t = \int_{-\infty}^{\infty} \frac{f(v) dv}{c + v}, \quad (6)$$

so the turbulence correction factor due to the Doppler effect is

$$F_{t,D} = \left[ \int_{-\infty}^{\infty} \frac{f(\beta) d\beta}{1 + \beta} \right]^{-1} \simeq 1 - \overline{\beta^2}, \quad (7)$$

where the second approximate equality follows from an expansion in  $\beta$ .

Analogously, the sound speed fluctuations  $c'$  give rise to a correction factor

$$F_{t,c} \simeq 1 - \overline{\delta^2}, \quad (8)$$

with  $\delta \equiv c'/c$ . In homogeneous isotropic turbulence scalar and vector fluctuations are uncorrelated by assumption,<sup>2</sup> so the two effects act independently, leading to a total turbulent correction of

$$F_t = 1 - (\overline{\beta^2} + \overline{\delta^2}). \quad (9)$$

In what follows, the overbars will be omitted, denoting by  $\beta$  and  $\delta$  the r.m.s. values.

The distribution of  $(\beta^2 + \delta^2)$  in the solar convective zone is shown in Figure 1. For this and all other figures, the mean thermal stratification of the solar envelope was taken from the model of Guenther *et al.* (1992), while for the turbulent velocities, the model of Unno, Kondo, and Xiong (1985) was used. It is clear that the effect of turbulence on sound propagation is limited to the uppermost few megameters.

The square of the effective sound speed  $C = F_{t,0}c$ , including the turbulent reduction, is displayed in Figure 2. A linear fit of the form

$$C^2 = mz \quad m = 10^{-4} \quad (10)$$

(dashed line) is found to be a tolerable representation that will greatly simplify the calculation and make an analytical solution possible. Such a linear dependence of the squared sound speed on depth is also characteristic of polytropic atmospheres with uniform gravitational acceleration. Here, however, we do not rely on any other assumption than equation (10). (Indeed, using a polytrope with the above value for  $m$  would lead to completely inappropriate acoustic cutoff frequencies and upper reflection depths.)

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velocity has a value between  $v$  and  $v + dv$ . Consequently, averages mean temporal averages throughout this paper.

<sup>2</sup> The correlation of a fluctuating scalar field  $\alpha$  and a fluctuating vector field  $\mathbf{u}$  is a mean vector field  $\langle \alpha \mathbf{u} \rangle$ . If non-vanishing, this field selects a preferred direction in space, contradicting the assumption of isotropy.

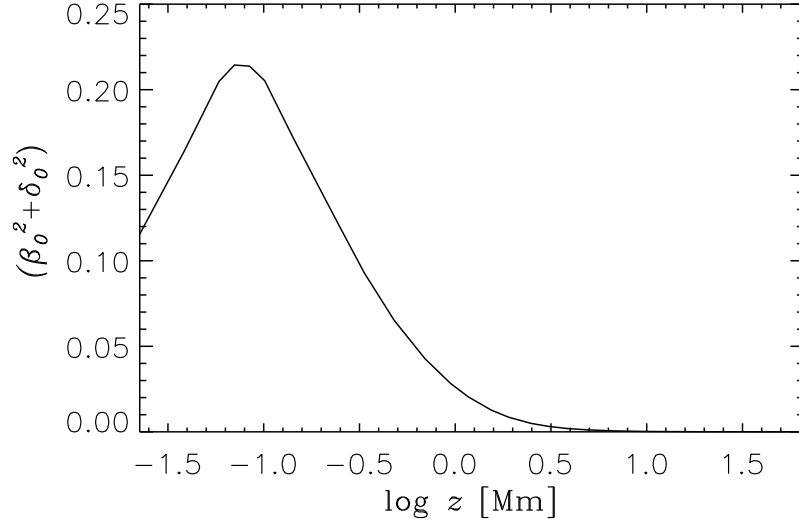


Figure 1. Mean square velocity fluctuations ( $\beta^2$ ) and sound speed fluctuations ( $\delta^2$ ), normalized by  $c^2$ , as a function of depth in the solar convective zone.  $z = 0$  corresponds to a continuum optical depth of unity. The index '0' on the ordinate refers to quiet sun values.

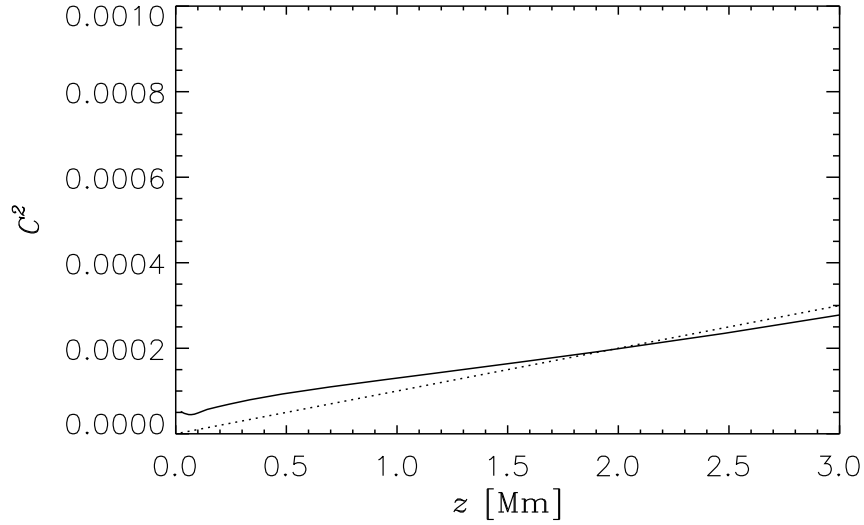


Figure 2. Square of the effective sound speed  $C = F_{t,0}c$ , including the turbulent reduction, as a function of depth in the solar convective zone. (Units are megameters for depth and Mm/s for the sound speed.)



In plage areas, magnetic fields will tend to suppress turbulent motions. On energetic grounds one expects this reduction to have the form

$$R \equiv \frac{\beta_1^2 + \delta_1^2}{\beta_0^2 + \delta_0^2} = \frac{1}{1 + (B/B_e)^2}, \quad (11)$$

where the subscripts 1 and 0 refer to plage and quiet sun values, respectively;  $B$  is the magnetic flux density,  $\rho$  is the mass density, and

$$B_e = (2\pi\rho\overline{v^2})^{1/2} \quad (12)$$

is the turbulent equipartition field strength.

The reasoning behind equation (11) goes as follows. The equipartition field  $B_e$  is the field strength where the magnetic energy density equals the kinetic energy density of turbulent motions. Then for  $B \gg B_e$  the strong field will effectively inhibit overturning turbulent motions ( $R \rightarrow 0$ ), while for  $B \ll B_e$  we expect that the field cannot significantly damp turbulence ( $R \rightarrow 1$ ), as its energy is not high enough to compensate the work done by the buoyancy forces driving the turbulent convection. Thus, we expect that the energy of the turbulence will be reduced by a factor  $1/R$  of order unity around  $B_e$ , i.e.  $1/R \sim E_{\text{turb}}/(E_{\text{turb}} + E_{\text{mag}})$ , as expressed in equation (11).

Thus, in a plage area the dispersion relation can be written as

$$\omega = FCk, \quad (13)$$

where  $C = F_t c$  is the quiet-sun sound speed (including turbulence effects), while

$$F = 1 + (1 - R)(\beta_0^2 + \delta_0^2) \quad (14)$$

is the correction factor due to the magnetic field. Clearly  $F > 1$ : the suppression of turbulence in active regions leads to less turbulent reduction of the sound speed there, i.e. sound waves will tend to propagate faster in plage than in quiet sun due to this effect.

Figure 3 shows  $(F - 1)$  as a function of depth for a presumed uniform field strength  $B = 400$  G. It is apparent that significant magnetic reduction of turbulence is limited to the uppermost 1 Mm approximately. This shallow layer of magnetically suppressed turbulence is observed in the form of abnormal granulation. We find that  $F(B, z)$  can be roughly approximated by the simple analytic function

$$F = 1 + f_1 \frac{(B/B_e)^2}{1 + (B/B_e)^2} e^{-\alpha z} (z/2)^{1/2} \quad f_1 = 1.8, \alpha = 6, \quad (15)$$

with  $z$  in megameters (dashed line on Fig. 3).

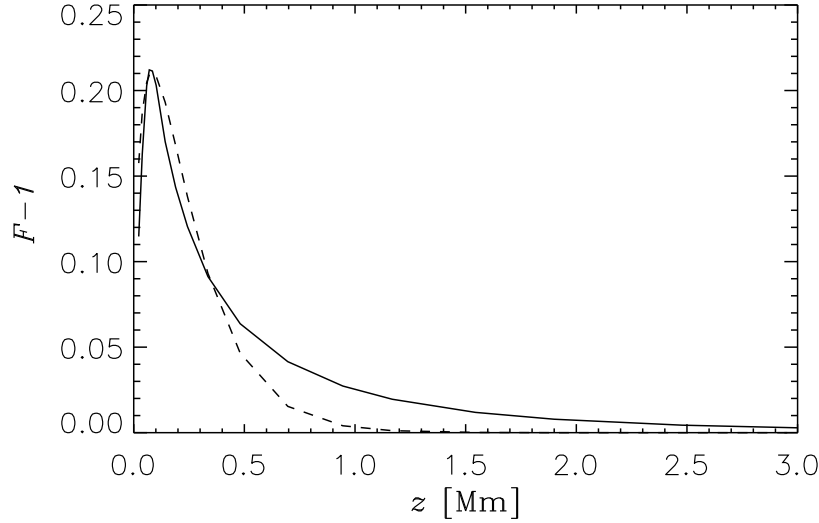


Figure 3. Magnetic correction to the sound speed in a plage area of field strength 400 G (*solid line*), and the analytic fit given in equation (15)

### 3. Results

Next, we substitute equations (10)–(15) into the ray equations (4) and (5). For the ray path we find that, for quiet sun, for fixed horizontal wave number  $k_x$  and frequency  $\omega$ , the lower reflection depth  $a$ , the travel distance  $d$ , and the travel time  $\tau$  are given by

$$a = \omega^2 / m k_x^2 \quad d = \pi a \quad \tau = \pi(m/a)^{1/2}, \quad (16)$$

(see e.g. Brüggén and Spruit, 2000 for details).

As the magnetic perturbation only extends down to a depth of  $z_0 \sim 1 \text{ Mm} \ll a$ , for the calculation of the perturbations in travel distance and travel time the integrals only need to be evaluated down to  $z_0$ . Expanding the integrands to first order in  $(F - 1)$ , and, upon substitution of equation (15), also in  $z/a$  results in

$$\Delta d \equiv d_{\text{plage}} - d_{\text{quiet sun}} = -f_1 \alpha^{-2} (2a)^{-1/2} [e^{-\alpha z_0} (1 + \alpha z_0) - 1], \quad (17)$$

with  $d$  and  $a$  in megameters. Evaluation of this formula shows that for  $a > 3 \text{ Mm}$ ,  $\Delta d < 10 \text{ km}$ . This effect is very small because in the shallow layer  $z < z_0$  the ray path is very close to vertical.

For the travel time reduction we find in an analogous manner

$$\Delta \tau \equiv \tau_{\text{quiet sun}} - \tau_{\text{plage}} = 2^{-3/2} f_1 m^{-1/2} \alpha^{-2} \frac{B^2/B_e^2}{1 + B^2/B_e^2} \times \quad (18)$$

$$\left\{ e^{-\alpha z_t} [2\alpha - a^{-1}(1 + \alpha z_t)] - e^{-\alpha z_0} [2\alpha - a^{-1}(1 + \alpha z_0)] \right\}.$$

Figures 4 and 5 show  $\Delta\tau$  as a function of upper reflection depth  $z_t$  and plage field strength  $B$ . The time shift shows only negligible dependence on the travel distance, as a simple consequence of the abnormal granulation layer depth  $z_0$  being small compared to the lower reflection depth  $a$ , so that the rays propagate nearly vertically in the affected region. This, in turn, implies that the *relative* travel time reduction decreases with travel distance, similar to the results of Brüggén and Spruit (2000). The overall magnitude of the travel time perturbation is of the order of 10 seconds. While this falls short of explaining the observed shifts of 20–60 seconds (Chou, 2000), the result shows that the sound speed perturbation introduced by abnormal granulation can explain a significant fraction of the measured time shifts.

The location of the upper turning point of the waves is a strong function of frequency, because of the variation of the acoustic cutoff frequency in the lower photosphere. We find that the time shift shows a marked frequency dependence (up to a factor 2 to 3 in the range 3 to 5 mHz), the travel time reduction being larger for higher frequencies. This is because higher frequency waves are reflected higher up in the photosphere and so spend a longer time in the layers affected by abnormal granulation.

As the suppression of turbulence, and, consequently, the perturbation of the sound speed, depends strongly on the magnetic energy density, we expect that the resulting time shifts should show a marked dependence on the value of the assumed plage field strength. This is borne out in Figure 5.

### 3.1. PHASE SHIFT VS. ENVELOPE SHIFT

As we have mentioned in Section 2.1 above, the ray approximation breaks down near the upper reflection point, where  $\omega_c$  is not negligible. The nearly vertically propagating wave is then described by the dispersion relation (1), resulting in different values for the group velocity  $c_g$  and the phase velocity  $c_p$ . From the dispersion relation it follows that

$$c_p c_g = c^2, \quad (19)$$

$$c_g/c_p = 1 - (\omega_c/\omega)^2. \quad (20)$$

Thus, the group velocity is less than the phase velocity, so that the wave train spends a longer time in the abnormal granulation layer than each wave crest does. This may be expected to give rise to a larger envelope time shift compared to the phase shift. Quantitatively,

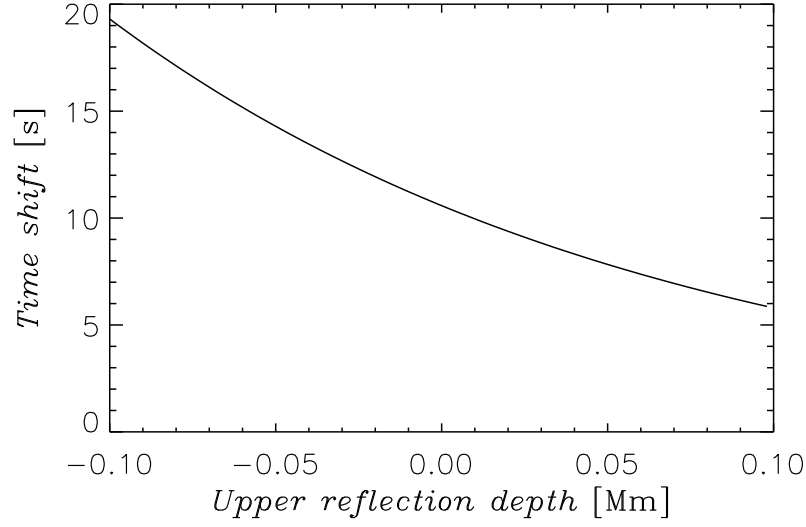


Figure 4. Travel time reduction in seconds for  $B = 400$  G as a function of the depth of the upper reflection, in the limit of large skip distance,  $d \gg z_0 \simeq 1$  Mm.

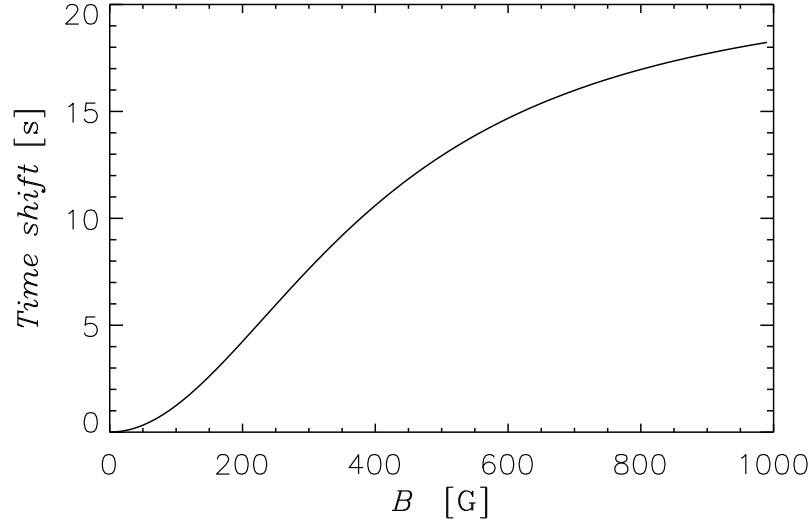


Figure 5. Travel time reduction for upper reflection at unit optical depth, as a function of field strength, in the limit of large skip distance,  $d \gg z_0 \simeq 1$  Mm.

the effect is rather sensitive to the exact profile of  $\omega_c$ , but a numerical estimate using the model of Guenther *et al.* (1992) shows that the resulting difference between phase and envelope time shifts may reach a factor of 2 to 3, which is comparable to the findings of Chou (2000).

### 3.2. FREQUENCY SHIFTS OF GLOBAL MODES

The upshot of the above results is that magnetic inhibition of turbulence in plage areas has the effect of increasing the effective sound speed in a shallow layer of depth  $z_0 \sim 1$  Mm. This, in turn, leads to a reduction  $\Delta\tau$  in sound wave travel times. The dependence of  $\Delta\tau$  on magnetic field strength  $B$  and frequency  $\nu$  is in qualitative agreement with the observations, but its amplitude is too low by a factor of 2 to 5. This suggests that the effect considered here is just one, albeit non-negligible contribution to the sound speed perturbation in plage areas. However, additional effects such as those considered by Zweibel and Bogdan (1986) should also be limited to a shallow layer, producing the same qualitative dependence.

A possibility to test these conclusions independently of the travel time measurements is offered by the determination of global mode frequency shifts in active regions from ring diagram analysis (Rajaguru, Basu, and Antia, 2001). Considering these shifts in the framework of our approach essentially corresponds to inverting the problem: instead of fixing  $k_x$  and  $\omega$  and looking for the skip distance  $d$  and the travel time, we now fix  $k_x$  and  $d$ , and look for  $\omega$ . Equating the wavelength to the skip distance we have  $d = 2\pi/k_x$ ; substituting this into equations (16) and introducing  $\nu = \omega/2\pi$  we have  $\nu = (m/a)^{1/2}/\pi = 1/\tau$ , as expected. From this

$$\Delta\nu/\nu = \Delta\tau/\tau = \nu\Delta\tau \quad (21)$$

As we have seen,  $\Delta\tau$  increases by a factor of 2 or so in the frequency range 3 to 5 mHz. Then equation (21) shows that  $\Delta\nu/\nu$  increases with  $\nu$  faster than linear (approximately quadratically). This is indeed confirmed by Figure 1 of Rajaguru, Basu, and Antia (2001) in the given frequency range. On the other hand, for frequencies below about 2.5 mHz the observed behavior changes to a growth slower than linear. This is most likely due to the non-negligible effect of spherical geometry on these low degree modes.

According to equation (21),  $\Delta\nu/\nu$  should show the same kind of dependence on the magnetic field strength as  $\Delta\tau$ , cf. Figure 5. This is indeed confirmed by Figure 2 of Rajaguru, Basu, and Antia (2001). (Note the different horizontal range on these two plots.)

The dependence of the relative frequency shifts on  $\nu$  and  $B$  is, then, qualitatively quite like what is observed. On the other hand, comparison of our Figure 5 with Figures 1 and 2 of Rajaguru et al. shows that the observed amplitude of the shift exceeds the predicted amplitude by about a factor of 2 for  $B = 100$  G. This is in accordance to the case of the travel time corrections discussed above, and suggests that another effect of similar depth dependence (most likely the mechanism proposed by Zweibel and Bogdan, 1986) is an important contributor to the sound speed perturbation in active regions.

#### 4. Discussion

In this paper we have considered the effect that the magnetic suppression of turbulence in plage areas should have on the propagation of acoustic waves. It has been found that any such effect is confined to a shallow surface layer of thickness below about 1 Mm, where sound waves propagate approximately vertically. Accordingly, the travel path of wave packets is essentially not modified by the effect considered. On the other hand, the weaker turbulence implies higher sound speeds, leading to shorter travel times. This time shift  $\Delta\tau$  (i.e. the reduction of the travel time) is independent of the travel distance, while it shows a characteristic dependence on the assumed plage field strength, plotted in Figure 5. As a consequence of the variation of the acoustic cutoff with height, and hence of the location of the upper turning point,  $\Delta\tau$  is expected to be significantly higher for higher frequency waves within the observed regime of 3–5 mHz. The lower group velocity near the upper reflection point further leads to an increased envelope time shift, as compared to the phase shift.  $p$ -mode frequencies in plage areas are expected to be higher by a corresponding amount,  $\Delta\nu/\nu = -\nu\Delta\tau$ .

These characteristics of the time and frequency shifts are in accordance with measurements, as reported e.g. by Chou (2000) and Rajaguru, Basu, and Antia (2001). However, the overall calculated amplitude of about ten seconds falls short of explaining the full observed time shifts, that exceed one minute in the case of envelope shift.

This is hardly surprising, as it is clear that magnetic suppression of turbulence is just one of a number of effects at play in plage areas that may influence sound propagation. Another prime candidate for explaining the time shifts is the direct effect of the fibril magnetic field on wave propagation, as considered by Zweibel and Bogdan (1986). Nevertheless, the overall qualitative agreement between our model and the observations suggests that any further important contribution to the time shifts should also be confined to a rather shallow surface layer.

The validity of our approach is certainly constrained by the simplifying assumptions made (ray approximation, frozen turbulence, isotropy). In particular, the effect of turbulent motions on acoustic waves has been considered by several authors (Stix, 2000; Murawski, 2003 and references therein) without making use of the ray approximation. These promising approaches, however, have not yet addressed the problem of the effect of a magnetic suppression of turbulence on wave propagation, studied in this paper. These more sophisticated models involve a larger number of unknown parameters and functions, such as degree of anisotropy or turbulence spectra. The effect of magnetic fields on these parameters is very poorly known, so their introduction would lead to a high degree of uncertainty into the problem at this stage.

Beside the direct effect of random velocity and magnetic fields on sound speed below the surface, a more indirect effect from *atmospheric* flows and magnetic fields can also be expected, as these effects change the boundary conditions for wave reflection.

Cunha, Brügger, and Gough (1998) suggested that acoustic wave scattering from a nearby sunspot could alone explain the apparent perturbed travel times found in plage areas, even in the absence of any magnetic field outside the spot. This effect could be separated from real physical effects, such as the one considered in the present paper, by a careful study of the systematic dependence of any time shift on the plage field strength. Exploratory work in this direction has been made by Hindman *et al.* (2000) and Rajaguru, Basu, and Antia (2001). Further systematic studies of this type will be needed until the relative contribution of different effects to the observed travel time shifts can be clarified.

### Acknowledgements

This research was partly funded by the UK Particle Physics & Astronomy Research Council under grant no. PP/X501812/1 and by the Royal Society International Joint Project RS15599. RE is grateful to M. K  ray for patient encouragement. K.P. acknowledges support from the ESMN network supported by the European Commission. R.E. and K.P. acknowledge support from the Hungarian National Science Research Fund under grant no. T043741.

### Appendix: The effect of a mean downflow

Contrary to what was assumed throughout this paper, turbulence in the solar convective zone is not homogeneous and isotropic. One potentially important consequence of this on sound wave propagation is that, owing to a nonvanishing correlation between velocity and density fluctuations (downflows are cooler and denser), a mean net downflow will arise, i.e.  $\langle v \rangle \neq 0$  even though  $\langle \rho v \rangle = 0$ . Numerical simulations indicate that immediately below unit optical depth the Mach number of this downflow may reach values of  $M \sim 0.1$ . As such a mean flow will advect the propagating wave, this might suggest to correct the r.h.s. of equation (7) to  $1 \pm M + \beta^2$ , the sign of  $M$  depending on whether the flow is parallel or antiparallel to the wave propagation. This may seem like a potentially important correction.

However, it should be taken into account that between two skips the wave front will first pass the shallow layers downwards (along the mean downflow), then upwards (against the mean downflow), so the net effect cancels to first order in  $M$ . For simplicity let us assume that the effective sound speed  $C$ , its magnetic perturbation  $\Delta C$  and the mean flow speed  $V$  are constant in the uppermost layer of thickness  $z_0$  where the magnetic field is dynamically important. Then the travel time reduction relative to the nonmagnetic case is

$$\begin{aligned} \Delta\tau &= \frac{z_0}{C+V} - \frac{z_0}{C+\Delta C+V} + \frac{z_0}{C-V} - \frac{z_0}{C+\Delta C-V} \\ &= z_0 \Delta C \left[ \frac{1}{(C+V)^2} + \frac{1}{(C-V)^2} \right] + \mathcal{O}[(\Delta C/C)^2] \end{aligned} \quad (22)$$

In the case with no mean flow ( $V = 0$ ) clearly  $\Delta\tau = \Delta\tau_0 \equiv 2z_0\Delta C/C^2$ . With this, series expansion of (22) yields

$$\Delta\tau/\Delta\tau_0 = 1 + 3M^2 + \mathcal{O}(M^3) \quad (23)$$

For  $M \equiv V/C \sim 0.1$  (a very generous upper limit, as in reality  $M$  decreases very fast with depth) this shows that the reduction of the travel time is increased by a few percents only compared to its value for homogeneous turbulence.

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